Sheet 7

Exercises for April 25

Sorry for being slightly late again and happy Easter.

Question 21. Maximal degree in the configuration model (Level A, 3 pts)
Show that the regularity assumption (6.13) from the lecture implies that

$$\max_{i\in[n]} d_i = o(\sqrt{n}).$$

Question 22. Random graph vs. configuration model (Level A, 3 pts)
Show Corollary 6.32 of the lecture: Let $\boldsymbol{d}^n = (d_1^n, \dots, d_n^n)$ be a sequence of degree sequences satisfying (6.13), and let $\mathcal{E}_n$ be collection of sets such that

$$P[\mathsf{CM}_n(\boldsymbol{d}^n) \in \mathcal{E}_n] \xrightarrow{n\to\infty} 1.$$

Then also

$$P[\text{uniform random simple graph with degree sequence } \boldsymbol{d}^n \in \mathcal{E}_n] \xrightarrow{n\to\infty} 1.$$

Question 23. Exercise (6.25) from the lecture (Level B, 3 pts)
Recall definitions (6.17) and (6.23) of M_n and $\tilde{M}_n$ from the lecture[1]. Observe that $M_n \neq \tilde{M}_n$ iff there is $i \neq j$ such that $x_{ij} \geq 3$. Use it to show that

$$\lim_{n\to\infty} P(M_n \neq \tilde{M}_n) = 0.$$

Question 24. Probability of i.i.d. sum being odd (Level B, 3 pts)
Assume that $(d_i)_{i\in[n]}$ is an i.i.d. sequence such that $P(d_i \text{ is even}) < 1$ and set $\ell_n = \sum_{i\in[n]} d_i$. Show that there is $c > 0$ such that

$$\left| P[\ell_n \text{ is odd}] - \frac{1}{2} \right| \leq \mathrm{e}^{-cn}.$$

Hint. Observe that $P[\ell_n \text{ is odd}] = \frac{1}{2}(1 - E[(-1)^{\ell_n}])$.

[1] In (6.17), there is a small mistake: M_n should be defined by $M_n = \frac{1}{2}\sum_i m_i$.